

aspects of the
l. In this case,
e Ising pressure
if J and $dJ/d\sigma$
are simply related
($69 kT_c$) and it
 $dJ/d\sigma < 0$. Let
ere n is a small
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er a small range

$$(11)$$

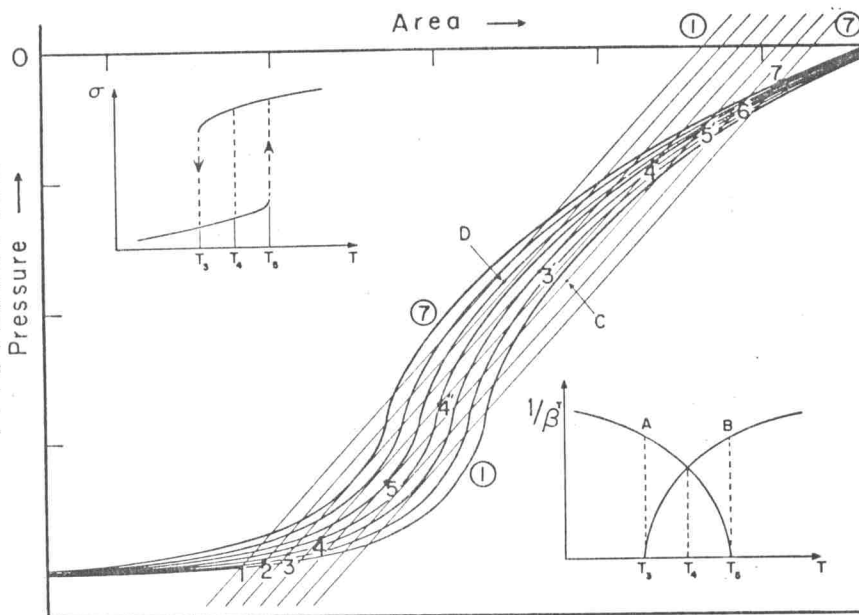
ts.

ire

an external ap-
ext = $p_{at} + p_I$. We
nal pressure, for
of p_I and $-p_{at}$
 $T_2 < \dots < T_6 < T_7$.
e isotherms will
external pressure
ied (that is, if
at of p_I). Now
 $p_{ext} = 0$. As the
 σ can increase
to 5 on Fig. 1),
becomes unstable
must be a first-
further heating
 σ_7 . However, on
crease smoothly
e the instability
rst-order change
ses smoothly on
sis loop near the
o in σ at T_5 on
t on Fig. 1. The
um width of this
anically unstable
a temperature T_4
4 equals that at
equilibrium would
nd no hysteresis.
ng or 4' and 3' on
t to show that a
etermining T_4 in

a schematic sketch
 $1/\beta^T$ in the critical
ched from below,
s to the value B
s. On cooling, as
ishes and jumps
f the system is in
t, $1/\beta^T$ never van-

FIG. 1. Behavior of a two-dimensional Ising model as a function of temperature at vanishing external pressure. The family of curves p_I were calculated at seven evenly-spaced temperatures from T_1 to T_7 . The family of straight lines $-p_{at}$ were drawn to represent a disordered lattice with typical compressibility and thermal expansion coefficients. The encircled numbers 1 and 7 indicate the spin and lattice isotherms at T_1 and T_7 . The insets represent schematically the temperature dependences of the area σ and of the reciprocal isothermal compressibility $1/\beta^T$.



ishes but has a singular point at T_4 . If the transition occurs in the metastable region but before the mechanical instability point is reached, $1/\beta^T$ will show hysteresis and discontinuities but does not vanish.

If an actual crystal behaves like this model, it is impossible to bring it arbitrarily close to a lambda point: a first-order transition occurs before the temperature reaches the theoretical critical temperature. Indeed, unless great care is taken to achieve true thermodynamic equilibrium, there are a range of temperatures (such as T_3 to T_5 in Fig. 1), where the properties depend on the history of the sample.

Constant Temperature

Let us look at the variation of area σ as a function of the applied pressure p_{ext} . Figure 2 shows, at a given temperature, the Ising pressure and the negative of the disordered-lattice pressure as functions of σ . At zero external pressure, the equilibrium point is at A , which corresponds to a largely disordered system. As the external pressure is increased, the ordering of the system increases and the area σ decreases smoothly until the pressure reaches a value equal to BB' at which mechanical instability occurs. The system spontaneously contracts to an area σ_C corresponding to Point C , which is the new equilibrium state under this external pressure of magnitude $p_3 = CC' = BB'$. A further increase in the external pressure causes a smooth decrease of the area and completes the ordering. If the pressure is now reduced, the system is mechanically stable until the area reaches the value $\sigma_D > \sigma_C$. At D the system is mechanically unstable and spontaneously expands to the value σ_B , the new equilibrium area under this pressure of magnitude $p_1 = EE' = DD'$. Again the possibility of hysteresis is predicted in a region which corresponds to metastable (or local) equilibrium. If

the system were in complete thermodynamic equilibrium a first-order transition without hysteresis would take place at pressure p_2 .

Constant Area

If the area is maintained constant by an applied pressure and not by rigid clamping, Inequality (10) is still valid. Therefore on each curve $p_I(T)$ for a given area, there is a forbidden zone in which the intersection of the isochores $-p_{at}(T)$ and $p_I(T)$ does not correspond to a stable state. On Fig. 3 are plotted several Ising isochores corresponding to areas $\sigma_1 < \sigma_2 < \dots < \sigma_6 < \sigma_7$. The

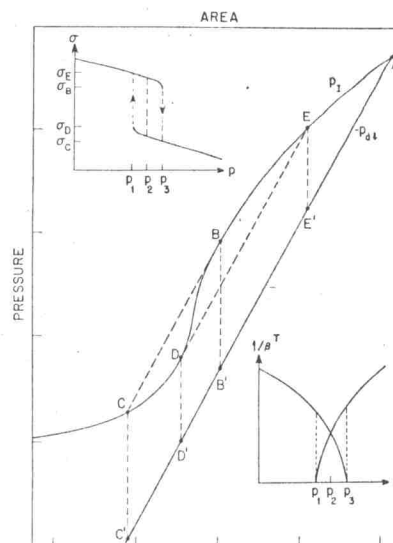


FIG. 2. Behavior of a two-dimensional Ising model as a function of pressure at constant temperature. The insets represent schematically the pressure dependence of the area σ and of the reciprocal isothermal compressibility $1/\beta^T$.